

BELLCOMM, INC.

955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

B70 04050

SUBJECT: An Improved Determination
of Optimal Moon-to-Earth
Trajectories for BCMASP -
Case 310

DATE: April 20, 1970
FROM: M. R. Kerr
R. J. Stern

ABSTRACT

A patched conic program (GNDHOM) has been written which generates a fuel optimal transearth trajectory from lunar parking orbit to earth landing. The trajectory may be fuel optimized with respect to longitude of earth landing, time of flight, and the time spent in lunar orbit after LM-CSM rendezvous.

All trajectories are constrained to enter the atmosphere at a given altitude with a specified entry angle and land within a specified geographic zone defined by longitude and latitude limits. In addition, a specified geographic return inclination limit and total mission duration limit are incorporated as constraints.

This method allows the determination of the optimal return trajectory for missions to lunar sites requiring highly inclined lunar parking orbits. Previous BCMASP optimization of the return trajectory relied on the assumption of low inclination lunar parking orbits.

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MEMORANDUM FOR FILE

INTRODUCTION

The nominal transearth injection (TEI) maneuver places the CSM on a trajectory from lunar parking orbit to a safe earth landing. The transearth trajectory is constrained to provide a suitable entry into the earth's atmosphere and a landing within a specified geographic zone defined by latitude and longitude limits. The trajectory must be such so as not to violate a maximum geographic return inclination. The TEI maneuver may be made any time after LM-CSM rendezvous as long as the resulting trajectory does not violate the specified maximum mission duration. The time in lunar parking orbit (LPO) after rendezvous may be specified or it may be optimized to produce minimal TEI fuel requirements.

Subject to the above requirements and constraints it is desirable to perform the TEI maneuver with minimum characteristic velocity (ΔV). The parameters that are available for optimization include longitude of earth landing, velocity azimuth at MSI exit, time of flight, and post-ascent time in lunar orbit. Previous analyses (References 1 and 2) provide for optimization with respect to exit velocity azimuth (ALF28P)* and earth landing longitude (ELN30P) within an allowable range. However, the optimization logic is based upon maximizing time of flight.** While this criteria is generally valid for low inclination lunar orbits it is not necessarily true for higher latitude sites and higher inclination lunar parking orbits. The program presented herein provides for optimization with respect to earth landing longitude, within a specified longitude band, that is based on a different selection criterion valid for any lunar parking orbit. It also provides the additional capability to optimize the TEI maneuver with respect to time of flight and time of departure from lunar orbit.

*Standard BCMASP variable names will be used throughout.

**Reference 2, see Program ANLSIS.

TECHNICAL APPROACH

A typical plot of TEI ΔV cost (DLV26P) versus time of flight from injection to landing (DTFL) for an extreme latitude landing site*** is illustrated in Figure 1. The existence of an optimal flight time is evident. This is the result of a lower plane change requirement at a higher energy (shorter flight time) return. For returns from low inclination lunar orbits the plane change requirements are usually small and the optimal return trajectory is governed by minimal energy or maximum time of flight.

The range of times of flight covered by the curves of Figure 1 correspond to a continuum of earth landing longitudes. Since Apollo ground rules call for a landing within a specified geographic zone, currently between 150° (ELNMAX) and 170° (ELNMIN) west longitude and $\pm 35^\circ$ latitude, only those bands illustrated in Figure 1 represent valid landing opportunities. Since the atmospheric entry point in geocentric inertial coordinates does not vary by more than a few degrees in longitude and since the reentry maneuver angle (THETAM) is fixed, these bands are spaced approximately 24 hours apart corresponding to the time interval during which the earth makes one revolution, thus bringing the landing zone into proper position for a spacecraft landing.

The following diagram graphically illustrates the order of optimization that is employed.

GNDHOM

Optimization with respect to post-rendezvous time in LPO

Optimization with respect to time of flight

FLYHOM

Optimization with respect to earth landing longitude

Optimization with respect to azimuth of the MSI exit velocity subject to limits determined from maximum inclination and maximum latitude constraints.

HOME

Targets transearth trajectory to given earth landing longitude

***Crater Tycho, $40^\circ 54'$ south, $11^\circ 21'$ west.

The optimization of both time of flight and earth landing longitude is controlled by the slope of the curve typically illustrated in Figure 1, that is the partial derivative of ΔV_{TEI} with respect to transearth time of flight, which is computed in HOME.

The GNDHOM time of flight optimization proceeds as follows. The longest flight time solution is computed first with the landing longitude equal to ELNMIN. The partial derivative of required TEI ΔV with respect to return time of flight (PD26DT) is computed in subroutine HOME as convergence to the solution occurs. If the sign of the partial derivative is negative then the maximum flight time solution is optimal and is returned. If the derivative is positive there exists the possibility of a lower ΔV solution at a lower flight time and times of flight approximately 24 hrs. earlier are investigated.

The sign of PD26DT also provides the criteria for selection of the optimal earth landing longitude in FLYHOM. If the sign of the partial is negative then the longitude corresponding to the longer flight time solution is selected (ELNMIN). If the sign is positive then ELNMAX is selected. It may be noted that the optimal longitude is at a boundary except for the case where a solution band exists at the minimum of the DLV26P vs. DTFL curve. However, in this region the slope of the curve is near zero and the selection of ELNMIN or ELNMAX causes a variation of only a few feet per second in DLV26P.

Optimization with respect to time in lunar orbit post-rendezvous is performed in an outer loop of GNDHOM. The number of lunar orbits post-rendezvous (REVLO2) is incremented from a minimum (REV2MN) to a maximum (REV2MX) in steps of three revolutions. For each value of REVLO2 considered, the optimal time of flight and associated DLV26P is computed. The REVLO2 which gives the minimum DLV26P is selected and returned as the optimal solution.

A flow chart of GNDHOM is presented in Appendix A and listings of GNDHOM and FLYHOM are given in Appendix B.

MODIFICATIONS TO EXISTING BCMASP PROGRAMS

Several modifications were made to the basic trans-earth injection optimization of BCMASP to implement the improved logic.

A. FLYHOM

The calculation of the optimum ALF28P was incorporated into a new subroutine, FLYHOM. The optimization of ALF28P within

specified limits is unchanged and is performed by calling subroutine OPTALF. These limits on ALF28P arise from the maximum return inclination and latitude constraints. The calculation of the maximum return inclination limits remains unchanged. The calculation of the ALF28P limits due to the latitude constraint was streamlined by eliminating the calculations for variable reentry maneuver angle. This new version greatly simplified the program logic and is valid as long as the landing latitude limits are greater than the maximum declination of the moon (28.7 degrees). The present limits are ± 35 degrees.

In addition to the modification of the ALF28P limit calculations the required logic for the selection of the optimal landing longitude has been incorporated into FLYHOM. This selection is based on the sign of the partial derivative, PD26DT, as described in the section labeled Technical Approach. A listing of FLYHOM is presented in Appendix B.

B. HOME

HOME targets the transearth trajectory using values for ALF28P and ELN30P from FLYHOM and values for REVLO2 and HRTMXP (maximum conic transearth flight time) from GNDHOM. HOME will allow a return time of flight from injection to entry (HRTEP) between HRTMXP+2 and HRTMXP-26 hours. Since landing solutions are spaced approximately 24 hours apart two solutions could exist within this range. This difficulty is avoided by the GNDHOM logic, which having obtained the approximate flight times calculates values of HRTMXP equal to the flight time + 12 hours. This insures that only one solution will exist in the range considered by HOME.

The calculation of the partial derivative PD26DT was incorporated in HOME. HOME consists of two nested iteration loops. The outer loop adjusts the geocentric velocity magnitude at MSI exit (VE28P) to achieve the desired landing longitude. The inner loop achieves convergence on the MSI position vector and time for a given value of VE28P. Since a change in landing longitude or VE28P really represents an adjustment in the time of flight the partial derivative is obtained by differencing the DLV26P obtained after convergence of the inner loop with the DLV26P of the previous iteration (different VE28P) and dividing by the difference in the time of flight. That is:

$$\frac{\partial \Delta V_{26}}{\partial t_{fl}} = PD26DT = \frac{\Delta V_{26_i} - \Delta V_{26_{i-1}}}{t_{fl_i} - t_{fl_{i-1}}}$$

where the subscripts i , $i-1$ denote two successive converged solutions for different values of $VE28P$. As convergence occurs $PD26DT$ approaches the slope of the ΔV cost for TEI versus time of flight curve (Figure 1).

SUMMARY

GNDHOM has been designed to calculate a minimum ΔV transearth trajectory from lunar parking orbit to a point within an earth landing zone. The optimization criteria is valid for returns from high and low inclination lunar parking orbits. The trajectory provides for a safe atmospheric entry and will not violate the maximum return inclination and mission duration constraints. The logic provides optimization with respect to velocity azimuth at MSI exit, earth landing longitude, time of flight and time in lunar orbit after LM-CSM rendezvous.

H.R. Kerr
M. R. Kerr

R. J. Stern
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Attachments

BELLCOMM, INC.

REFERENCES

1. R. J. Amman, Mission Analysis and Open-Loop Trajectory Targeting Theory for the Bellcomm Apollo Simulation Program - Bell Telephone Laboratories, Technical Memorandum 66-4264-2, January 10, 1966.
2. Bellcomm Apollo Simulation Program Revisions, October 14, 1966.

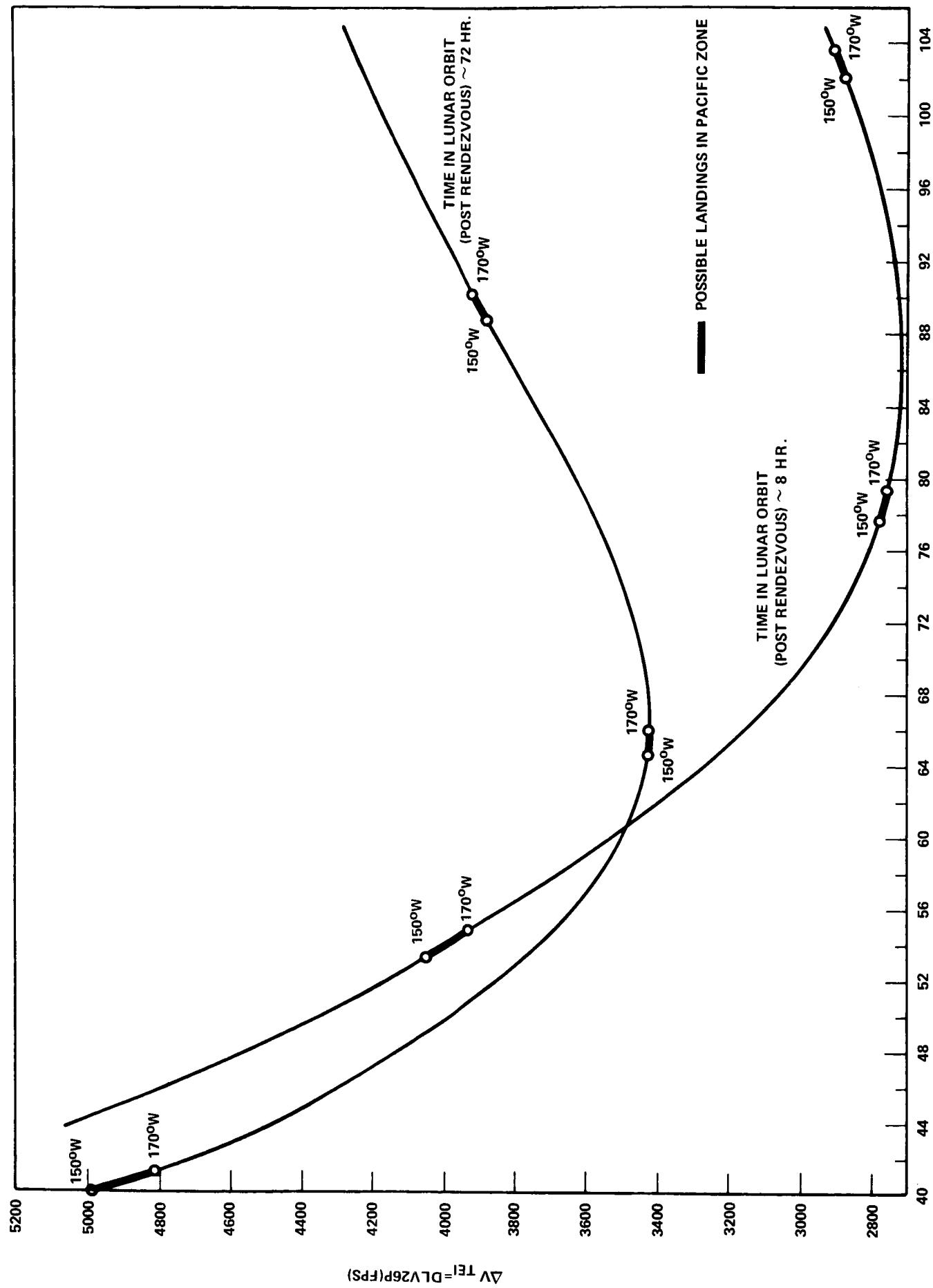


FIGURE 1 - TEI Δ vs TIME OF FLIGHT FOR A HIGH LATITUDE SITE.

APPENDIX A

GNDHOM

INPUT

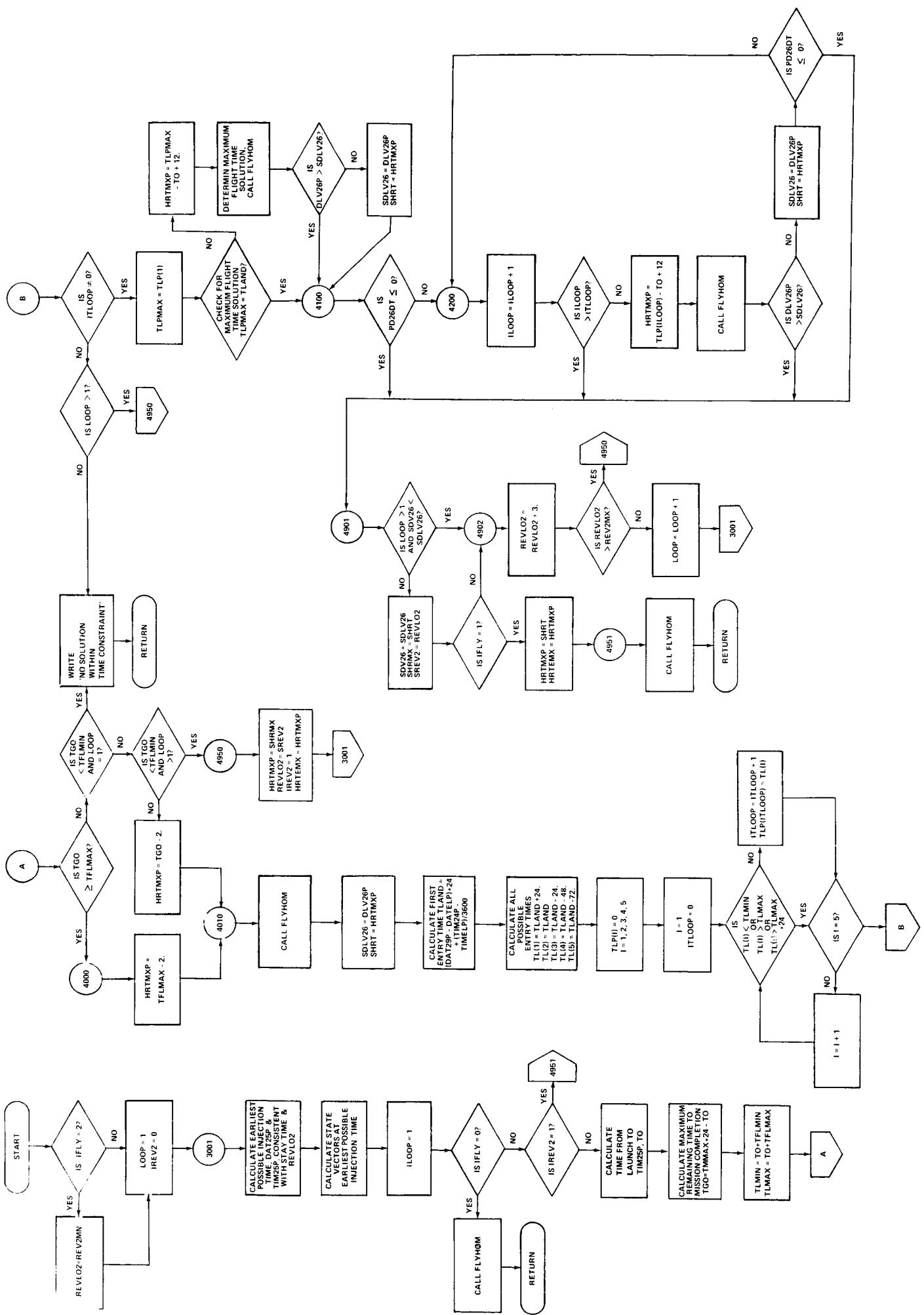
REV2MN	Minimum number of revolutions in lunar orbit, post-rendezvous
REV2MX	Maximum number of revolutions in lunar orbit, post-rendezvous
TMMAX	Maximum mission duration (days)
TFLMIN	Minimum transearth time of flight (hr.)
TFLMAX	Maximum transearth time of flight (hr.)
IFLY	Optimization flag - 0 ALF28P, ELN30P 1 ALF28P, ELN30P, HRTEP 2 ALF28P, ELN30P, HRTEP, REVLO2.
REVLO2	Number of revolutions in lunar orbit, post- rendezvous (needed if IFLY = 0 or 1)
DATELP	Julian date of launch
TIMELP	Universal time of launch (sec.)
DAT29P	Julian date of reentry
TIME29P	Universal time of reentry (sec.)
DAT23P	Julian date of lunar landing
TIM23P	Universal time of lunar landing (sec.)
TSTAY	Surface stay time (sec.)
PERLOP	Period of conic lunar orbit (sec.)
DEGCIR	360 (degrees)
HMLOPX(3)	Unit lunar orbit angular momentum vector
RM23PX(3)	Selenocentric vector to position of CSM on conic trajectory at approximate time of LM landing (ft.)

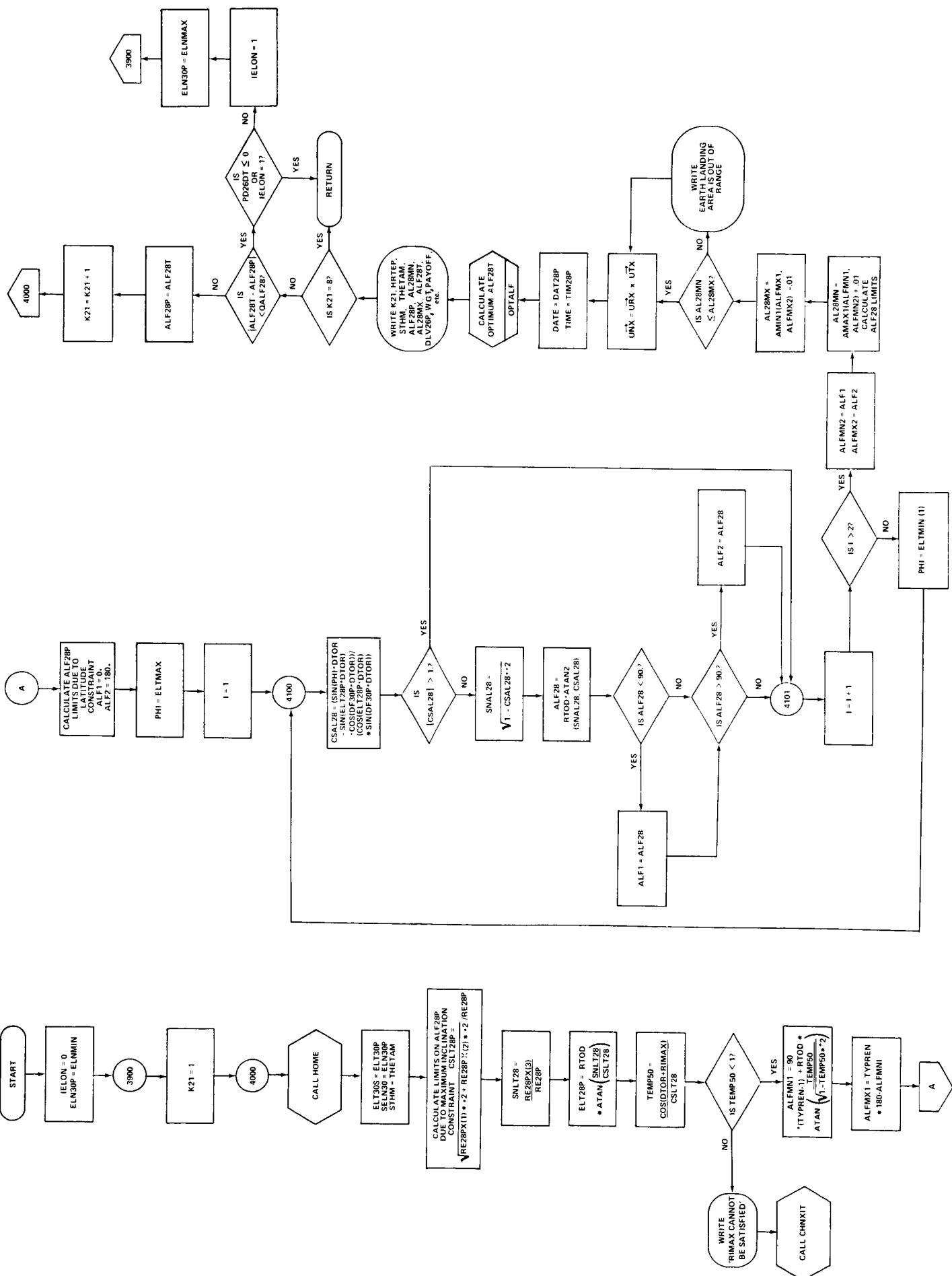
INPUT (continued)

VM23PX(3) CSM selenocentric velocity vector on conic trajectory at approximate time of LM landing (fps)

OUTPUT

REVLO2 Number of revolutions in lunar orbit post-rendezvous (only if IFLY = 2)
HRTMXP Maximum conic transearth flight time (hr.)
HRTEMX Maximum allowed transearth flight duration (hr.)





APPENDIX B

Program Listing

C SUBROUTINE GNDHOM 000100S1
C 000200S1
C 000300S1
C 000400S1
C IF(IFLY.EQ.2) REVLO2=REV2MIN 000500S1
LOOP=1 000600S1
IREV2=0 000700S1
3001 CALL ADDT(DAT23P,TIM23P,0.,TSTAY+(REVLO2+1.)*PERLOP,DAT25P,TIM25P) 000800S1
TEMP51=DEGCIR*(REVLO2-AINT(REVLO2)) 000900S1
CALL ROTATE (RM23PX,HMLOPX,TEMP51,RM25PX) 001000S1
CALL ROTATE (VM23PX,HMLOPX,TEMP51,VM25PX) 001100S1
ILOOP=1 001200S1
C 001300S1
C 001400S1
C TRANSEARTH TRAJECTORY 001500S1
C 001600S1
IF(IFLY.NE.0) GO TO 3999 001700S1
CALL FLYHOM 001800S1
RETURN 001900S1
3999 IF(IREV2.EQ.1) GO TO 4951 002000S1
T0=(DAT25P-DATELP)*24.+ (TIM25P-TIMELP)/3600.
TGU=TMMAX*24.-T0 002100S1
TLMIN=TU+TFLMIN 002200S1
TLMAX=TU+TFLMAX 002300S1
IF(TGU.GE.TFLMAX) GO TO 4000 002400S1
IF(TGU.LT.TFLMIN.AND LOOP.EQ.1) GO TO 4999 002500S1
IF(TGU.LT.TFLMIN.AND LOOP.GT.1) GO TO 4950 002600S1
HRTMXP=TGU-2.
GO TO 4010 002700S1
002800S1
002900S1
4000 HRTMXP=TFLMAX-2.
4010 CALL FLYHOM 003000S1
SDLV26=DLV26P 003100S1
SHRT=HRTMXP 003200S1
TLAND=(DAT29P-DATELP)*24.+ (TIM29P-TIMELP)/3600.
TL(1)=TLAND+24.
TL(2)=TLAND 003500S1
TL(3)=TLAND-24.
TL(4)=TLAND-48.
TL(5)=TLAND-72.
DO 4020 I=1,5 003600S1
003700S1
003800S1
003900S1
004000S1
4020 TLP(I)=0.
ITLOOP=0 004100S1
DO 4030 I=1,5 004200S1
004300S1
IF(TL(I).LT.TLMIN.OR.TL(I).GT.TLMAX.OR.TL(I).GT.TMMAX*24.) GO TO 004400S1
14030 004500S1
ITLOOP=ITLOOP+1 004600S1
TLP(ITLOOP)=TL(I) 004700S1
4030 CONTINUE 004800S1
IF(ITLOOP.NE.0) GO TO 4031 004900S1
IF(LOOP.GT.1) GO TO 4950 005000S1
GO TO 4999 005100S1
4031 TLPMAX=TLP(1) 005200S1
IF(TLPMAX.EQ.TLAND) GO TO 4100 005300S1
HRTMXP=TLPMAX-T0+12. 005400S1

CALL FLYHOM	005500S1
IF(DLV26P.GT.SDLV26) GO TO 4100	005600S1
SDLV26=DLV26P	005700S1
SHRT=HRTMXP	005800S1
4100 IF(PD26DT.LE.0.) GO TO 4901	005900S1
4200 ILOOP=ILOOP+1	006000S1
IF(ILOOP.GT.ITLOOP) GO TO 4901	006100S1
HRTMXP=TLP(ILOOP)-T0+12.	006200S1
CALL FLYHOM	006300S1
IF(DLV26P.GT.SDLV26) GO TO 4901	006400S1
SDLV26=DLV26P	006500S1
SHRT=HRTMXP	006600S1
IF(PD26DT.LE.0.) GO TO 4901	006700S1
GO TO 4200	006800S1
4901 IF(LOOP.GT.1.AND.SDV26.LT.SDLV26) GO TO 4902	006900S1
SDV26=SDLV26	007000S1
SHRMX=SHRT	007100S1
SREV2=REVL02	007200S1
IF(IFLY.EQ.1) GO TO 4952	007300S1
4902 REVL02=REVL02+3.	007400S1
IF(REVL02.GT.REV2MX) GO TO 4950	007500S1
LOOP=LOOP+1	007600S1
GO TO 3001	007700S1
4950 HRTMXP=SHRMX	007800S1
REVL02=SREV2	007900S1
IREV2=1	008000S1
HRTEMX=HRTMXP	008100S1
GO TO 3001	008200S1
4952 HRTMXP=SHRT	008300S1
HRTEMX=HRTMXP	008400S1
4951 CALL FLYHOM	008500S1
RETURN	008600S1
C	008700S1
4999 WRITE(6,4998)	008800S1
4998 FORMAT(/' NO SOLUTION WITH IN FLIGHT TIME CONSTRAINT ')	008900S1
RETURN	009000S1

```

C          SUBROUTINE FLYHOM          000100S1
C          TRANSEARTH TRAJECTORY   000200S1
C          TELON=U                000300S1
C          ENL30P=ELNMIN          000400S1
C          K21=1                  000500S1
C          3900 CALL HOME           000600S1
C          ELT30S=ELT30P          000700S1
C          SELN30=ELN30P          000800S1
C          STHM=THETAM           000900S1
C          001000S1
C          001100S1
C          001200S1
C          001300S1
C          001400S1
C          001500S1
C          001600S1
C          CALCULATE LIMITS ON ALF28P DUE TO MAXIMUM INCLINATION 001700S1
C          CSLT28=SQRT( RE28PX(1)**2+RE28PX(2)**2)/RE28P      001800S1
C          SNLT28=RE28PX(3)/RE28P                            001900S1
C          ELT28P=R10D*ATAN(SNLT28/CSLT28)                  002000S1
C          TEMP50=COS(DTOR*RIMAX)/CSLT28                   002100S1
C          IF(TEMP50.LT.1.) GO TO 4001                      002200S1
C          WRITE(6,9005)                                     002300S1
C          9005 FORMAT(1H0,5X,26HRIMAX CAN NOT BE SATISFIED)    002400S1
C          CALL CHNXIT                                      002500S1
C          4001 ALFMN1=90.*(TYPREN-1.)+RTOD*ATAN(TEMP50/SQRT(1.-TEMP50**2)) 002600S1
C          ALFMX1=TYPREN*180.-ALFMN1                         002700S1
C          CALCULATE LIMITS ON ALF28P DUE TO LATITUDE CONSTRAINT 002800S1
C          ALF1=0.                                           002900S1
C          ALF2=180.                                         003000S1
C          PHI=ELTMAX                                       003100S1
C          I=1                                             003200S1
C          4100 CSAL28=(SIN(PHI*DTOR)-SIN(ELT28P*DTOR)*COS(DF30P*DTOR))/(COS(ELT28P*DTOR)*SIN(DF30P*DTOR)) 003300S1
C          IF(ABS(CSAL28).GT.1.) GO TO 4101                 003400S1
C          SNAL28=SQRT(1.-CSAL28**2)                      003500S1
C          ALF28=RTOD*ATAN2(SNAL28,CSAL28)                 003600S1
C          IF(ALF28.LT.90.) ALF1=ALF28                     003700S1
C          IF(ALF28.GT.90.) ALF2=ALF28                     003800S1
C          4101 I=I+1                                         003900S1
C          IF(I.GT.2) GO TO 4102                           004000S1
C          PHI=ELTMIN(1)                                    004100S1
C          GO TO 4100                                       004200S1
C          4102 ALFMN2=ALF1                                 004300S1
C          ALFMX2=ALF2                                 004400S1
C          004500S1
C          CALCULATE LIMITS ON ALF28P                      004600S1
C          004700S1
C          004800S1
C          AL28MN=AMAX1(ALFMN1,ALFMN2)+.01               004900S1
C          AL28MX=AMIN1(ALFMX1,ALFMX2)-.01               005000S1
C          IF(AL28MN.LE.AL28MX) GO TO 4199              005100S1
C          WRITE(6,7001)                                    005200S1
C          7001 FORMAT(1H0,5X,34HEARTH LANDING AREA IS OUT OF RANGE) 005300S1
C          005400S1

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C      CALCULATE OPTIMUM ALF28P          005500S1
C
4199 CALL CROSS(URX,UTX,UNX,0)          005600S1
     DATE=DAT28P                         005700S1
     TIME=TIM28P                         005800S1
     CALL OPTALF(ALF28P,AL28MN,AL28MX,VE28TP,VE28RP,HMLUPX,ALF28T) 005900S1
C
5000 WRITE(6,5001)K21,HRTEP,STHM,THETAM, ALF28P,AL28MN,AL28MX,ALF28T,DL006200S1
     1V26P,WGT,PAYOFF,RAMIN1,RAMAX1,ELT30S,SELN30,                         006000S1
     2RAMIN2,RAMAX2,ELT30P,ELN30P,RELNMMN,RELNMX,RA                         006100S1
     WRITE(6,5002) PD26DT,REVL02,HRTMXP                                     006200S1
5002 FORMAT(6X,'PD26DT',2X,E12.6,8X,'REVL02',2X,F12.6,8X,'HRTMXP',2X,F1006600S1
     12.4)                                         006300S1
5001 FORMAT(1H0,5X,3HK21,15X,I2,8X,5HHRTEP,3X,F12.6,8X,OHTHETAM,2X,F12.006800S1
     1O,8X,5HHTHOPT,3X,F12.6/6X,6HALF28P,2X,F12.6,8X,OHAL28MN,2X,F12.6,8X0076900S1
     2,6HAL28MX,2X,F12.6,8X,OHALF28T,2X,F12.6/6X,6HDLV26P,2X,F12.4,8X,3H007000S1
     3wGT,5X,F12.3,8X,OHPAYOFF,2X,F12.4/                                007100S1
     4oX,6HRAMIN1,2X,F12.6,8X,6HRAMAX1,2X,F12.0,8X,6HELT30P,2X,F12.6, 007200S1
     58X,6HELN30P,2X,F12.0/6X,6HRAMIN2,2X,F12.6,8X,6HRAMAX2,2X,F12.6, 007300S1
     68X,6HELT0PT,2X,F12.6,8X,OHELNOPT,2X,F12.0/6X,6HRELNMMN,2X,        007400S1
     7F12.6,8X,6HRELNMX,2X,F12.6,8X,2HRA,6X,F12.6)                      007500S1
5500 IF(K21.EQ.8) RETURN               007600S1
     IF(ABS(ALF28T-ALF28P).LT.QALF28) GO TO 5510 007700S1
     ALF28P=ALF28T                         007800S1
     K21=K21+1                           007900S1
     GO TO 4000                           008000S1
5510 IF(PD26DT.LE.0..OR.IELON.EQ.1) RETURN 008100S1
     IELON=1                            008200S1
     ELN30P=ELNMAX                        008300S1
     GO TO 3900                           008400S1
     END                                008500S1

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